

Heavy hybrid mesons in the QCD sum rule

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We study the spectra of the hybrid mesons containing one heavy quark ($q\bar{Q}g$) within the framework of QCD sum rules in the heavy quark limit. The derived sum rules are stable with the variation of the Borel parameter within their corresponding working ranges. The extracted binding energy for the heavy hybrid doublets $H(S)$ and $M(T)$ is almost degenerate. We also calculate the pionic couplings between these heavy hybrid and the conventional heavy meson doublets using the light-cone QCD sum rule method. The extracted coupling constants are rather small as a whole. With these couplings we make a rough estimate of the partial widths of these pionic decay channels.

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I. INTRODUCTION

Hadron states which can not be accommodated in the conventional quark model have attracted much interest over the past few decades, partly due to the great success of quark model in the classification of hadrons and the calculation of hadronic parameters. Theoretically, the existence of these unconventional hadrons may be allowed by Quantum Chromodynamics (QCD), the widely accepted fundamental theory of the strong interaction.

These unconventional hadrons include multi-quark states ($qq\bar{q}\bar{q}$, $qqq\bar{q}$, \dots), glueballs (gg , ggg , \dots), and hybrids ($q\bar{q}g$). Some of them are totally “exotic”, namely their J^{PC} quantum numbers are excluded by the conventional quark model. A straightforward analysis of J^{PC} reveals that 0^{--} , 0^{+-} , 1^{-+} , 2^{+-} , \dots are the so called exotic ones. Several mesons with exotic $J^{PC} = 1^{-+}$, e.g. $\pi_1(1400)$ [1], $\pi_1(1600)$ [2], have been reported in recent years. These states are usually interpreted to be hybrid mesons. The 1^{-+} states have been studied in the framework of QCD sum rules in several works, including their masses [3] and decay properties [4]. They were also investigated extensively in other theoretical schemes, such as Lattice QCD, the flux tube model, and AdS/QCD etc.

If the above mentioned hybrid mesons exist, there should also be hybrid mesons containing one heavy quark ($q\bar{Q}g$) and heavy hybrid quarkonium ($Q\bar{Q}g$), although the J^{PC} quantum number of the former is not exotic. The hybrid mesons containing one heavy quark and heavy hybrid quarkonium have been studied in [5]. The masses of the hybrid quarkonia were calculated in the heavy quark limit [6]. The masses and the pionic couplings to conventional heavy mesons of the hybrids containing one heavy quark were studied in Ref. [7]. In the present work, we study the hybrid mesons containing one heavy quark in the framework of heavy quark effective theory (HQET) [8], in which the expansion is performed in terms of $1/m_Q$, where Q is the heavy quark involved. At the leading order of $1/m_Q$, the HQET Lagrangian respects the heavy quark flavor-spin symmetry, therefore heavy hadrons form a series of degenerate doublets. The two members in a doublet share the same quantum number j_l , the angular momentum of the light components. The two $j_l = \frac{1}{2}$ S -wave conventional heavy mesons form a doublet ($0^-, 1^-$) denoted as H and the $j_l = \frac{1}{2}/\frac{3}{2}$ P -wave doublets ($0^+, 1^+$)/($1^+, 2^+$) are denoted as S/T . We denote the $j_l = \frac{3}{2}/\frac{5}{2}$ D -wave doublets ($1^-, 2^-$)/($2^-, 3^-$) as M/N . As far as the heavy hybrid containing one heavy quark are concerned, the two $j_l = \frac{1}{2}$ doublets with $P = +$ and $P = -$ are denoted as S^h and H^h , respectively. Similarly, the two $j_l = \frac{3}{2}$ doublets with $P = +$ and $P = -$ are denoted as T^h and M^h , respectively.

We calculate the binding energy and decay constants of these hybrid doublets using Shifman-Vainshtein-Zakharov (SVZ) sum rules [9]. After performing the operator product expansion (OPE) of the T product of two interpolating currents, we obtain sum rules which relate the binding energy and decay constants of corresponding hybrid mesons to expressions containing vacuum condensates parameterizing the QCD nonperturbative effect. The nonperturbative method used to calculate the pionic couplings between heavy hybrid mesons and conventional heavy mesons is the light-cone QCD sum rules (LCQSR) [10]. Now the OPE of the T product of two interpolating currents sandwiched between the vacuum and an hadronic state is performed near the light-cone rather than at a small distance as in SVZ

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sum rules, and the QCD nonperturbative effects are included in the light-cone distribution amplitudes of the pion state.

The paper is organized as follows. We construct the interpolating currents for the doublets D^h ($D = H/S/T/M$) in Sec. II. We derive the sum rules for the binding energy and decay constants for these doublets in Sec. III. The sum rules for their pionic couplings to the doublets H and S are derived in Sec. IV. The last section is a short summary. The light cone distribution amplitudes of the pion which are employed in the present calculation are collected in the appendix.

II. INTERPOLATING CURRENTS

We adopt the following interpolating currents for the doublets H^h and M^h :

$$\begin{aligned} J_{H_0^h}^\dagger &= \sqrt{\frac{1}{2}} \bar{h}_v i g_s \gamma_5 \sigma_t \cdot G q, \\ J_{H_1^h}^{\dagger\alpha} &= \sqrt{\frac{1}{2}} \bar{h}_v i g_s \gamma_t^\alpha \sigma_t \cdot G q, \\ J_{M_1^h}^{\dagger\alpha} &= \bar{h}_v g_s \left[3 G_t^{\alpha\beta} \gamma_\beta + i \gamma_t^\alpha \sigma_t \cdot G \right] q, \\ J_{M_2^h}^{\dagger\alpha_1\alpha_2} &= \sqrt{\frac{3}{2}} \bar{h}_v g_s \gamma_5 \left[G_t^{\alpha_1\beta} \gamma_\beta \gamma_t^{\alpha_2} + G_t^{\alpha_2\beta} \gamma_\beta \gamma_t^{\alpha_1} - \frac{2}{3} i g_t^{\alpha_1\alpha_2} \sigma_t \cdot G \right] q, \end{aligned} \quad (1)$$

where $G_{\alpha\beta} = G_{\alpha\beta}^m \lambda^n / 2$ and $h_v(x) = e^{im_Q v \cdot x} \frac{1+\not{v}}{2} Q(x)$ is the heavy quark field with 4-velocity v . The subscript t is used to denote the transverseness of the corresponding Lorentz tensors to v , namely

$$\begin{aligned} \gamma_t^\alpha &= \gamma^\alpha - \not{v} v^\alpha, \\ g_t^{\alpha\beta} &= g^{\alpha\beta} - v^\alpha v^\beta, \\ \sigma_t^{\alpha\beta} &= \sigma^{\alpha\beta} - \sigma^{\alpha\mu} v_\mu v^\beta - \sigma^{\mu\beta} v_\mu v^\alpha, \\ G_t^{\alpha\beta} &= G^{\alpha\beta} - G^{\alpha\mu} v_\mu v^\beta - G^{\mu\beta} v_\mu v^\alpha. \end{aligned} \quad (2)$$

The overlapping amplitudes between the above currents and the corresponding hybrids are defined as

$$\begin{aligned} \langle 0 | J_{H_0^h}(0) | H_0^h(v) \rangle &= f_{H_0^h}, \\ \langle 0 | J_{H_1^h}^\alpha(0) | H_1^h(v, \lambda) \rangle &= f_{H_1^h} \eta_{H_1^h}^\alpha(v, \lambda), \\ \langle 0 | J_{M_1^h}^\alpha(0) | M_1^h(v, \lambda) \rangle &= f_{M_1^h} \eta_{M_1^h}^\alpha(v, \lambda), \\ \langle 0 | J_{M_2^h}^{\alpha_1\alpha_2}(0) | M_2^h(v, \lambda) \rangle &= f_{M_2^h} \eta_{M_2^h}^{\alpha_1\alpha_2}(v, \lambda), \end{aligned} \quad (3)$$

where $\eta(v, \lambda)$ is the polarization tensor of the corresponding heavy hybrid. Apparently these polarization tensors are traceless, symmetric to their Lorentz index and transversal to v : $\eta_{\alpha\alpha_2 \dots \alpha_n} v^\alpha = 0$. Furthermore, summations on λ give the following projection operators:

$$\begin{aligned} \sum_\lambda \eta^\alpha(v, \lambda) \eta^\beta(v, \lambda) &= -g_t^{\alpha\beta}, \\ \sum_\lambda \eta^{\alpha_1\alpha_2}(v, \lambda) \eta^{\beta_1\beta_2}(v, \lambda) &= \frac{1}{2} g_t^{\alpha_1\beta_1} g_t^{\alpha_2\beta_2} + \frac{1}{2} g_t^{\alpha_1\beta_2} g_t^{\alpha_2\beta_1} - \frac{1}{3} g_t^{\alpha_1\alpha_2} g_t^{\beta_1\beta_2}. \end{aligned} \quad (4)$$

The angular momentum of the light components j_l of the heavy hybrids h that couple to the current $J_{M_1^h}^{\dagger\alpha}$ must be $3/2$. To illuminate this point, we first notice that the 4-velocity of the light quark q of h should be v , namely $\not{v} q = q$. This leads to $\not{v} R^\alpha = -R^\alpha$ and $\gamma_\alpha R^\alpha = 0$, where $R^\alpha \equiv \left[G_t^{\alpha\beta} \gamma_\beta + \frac{i}{3} \gamma_t^\alpha \sigma_t \cdot G \right] q$. In other words, R^α is a Rarita-Schwinger spinor describing a spin $3/2$ particle when the 4-velocity of q is v .

Now it is straightforward to give the interpolating currents for the doublets S^h and T^h by adding γ_5 to the currents in Eq. (1):

$$J_{S_0^h}^\dagger = \sqrt{\frac{1}{2}} \bar{h}_v i g_s \sigma_t \cdot G q,$$

$$\begin{aligned}
J_{S_1^h}^{\dagger\alpha} &= \sqrt{\frac{1}{2}} \bar{h}_v i g_s \gamma_5 \gamma_t^\alpha \sigma_t \cdot G q, \\
J_{T_1^h}^{\dagger\alpha} &= \bar{h}_v g_s \gamma_5 \left[3 G_t^{\alpha\beta} \gamma_\beta + i \gamma_t^\alpha \sigma_t \cdot G \right] q, \\
J_{T_2^h}^{\dagger\alpha_1\alpha_2} &= \sqrt{\frac{3}{2}} \bar{h}_v g_s \left[G_t^{\alpha_1\beta} \gamma_\beta \gamma_t^{\alpha_2} + G_t^{\alpha_2\beta} \gamma_\beta \gamma_t^{\alpha_1} - \frac{2}{3} i g_t^{\alpha_1\alpha_2} \sigma_t \cdot G \right] q.
\end{aligned} \tag{5}$$

The corresponding overlapping amplitudes and projection operators can be defined similarly to Eq. (18) and (4), respectively.

III. BINDING ENERGY

To derive the sum rules for the binding energy for the doublets H^h and M^h , we consider the following correlation functions:

$$\begin{aligned}
i \int d^4x e^{ik \cdot x} \langle 0 | T \{ J_{H_0^h}(x) J_{H_0^h}^\dagger(0) \} | 0 \rangle &= \Pi_{H_0^h}(\omega), \\
i \int d^4x e^{ik \cdot x} \langle 0 | T \{ J_{H_1^h}^\alpha(x) J_{H_1^h}^{\dagger\beta}(0) \} | 0 \rangle &= -g_t^{\alpha\beta} \Pi_{H_1^h}(\omega), \\
i \int d^4x e^{ik \cdot x} \langle 0 | T \{ J_{M_1^h}^\alpha(x) J_{M_1^h}^{\dagger\beta}(0) \} | 0 \rangle &= -g_t^{\alpha\beta} \Pi_{M_1^h}(\omega), \\
i \int d^4x e^{ik \cdot x} \langle 0 | T \{ J_{M_2^h}^{\alpha_1\alpha_2}(x) J_{M_2^h}^{\dagger\beta_1\beta_2}(0) \} | 0 \rangle &= \left[\frac{1}{2} g_t^{\alpha_1\beta_1} g_t^{\alpha_2\beta_2} + \frac{1}{2} g_t^{\alpha_1\beta_2} g_t^{\alpha_2\beta_1} - \frac{1}{3} g_t^{\alpha_1\alpha_2} g_t^{\beta_1\beta_2} \right] \Pi_{M_2^h}(\omega),
\end{aligned} \tag{6}$$

where $\omega = 2k \cdot v$. The correlation functions for doublets S^h and T^h are similar to that of H^h and M^h , respectively.

At the quark level ($\omega \ll 0$), it is convenient to calculate the above correlators in coordinate space by OPE at a short distance $x \rightarrow 0$. The fourier transformation to the momentum space is straightforward after employing the heavy quark propagator in the infinity quark mass limit $m_Q \rightarrow \infty$:

$$\langle 0 | T \{ h_v(x) \bar{h}_v(0) \} | 0 \rangle = \frac{1 + \not{v}}{2} \int_0^\infty dt \delta(x - vt). \tag{7}$$

The quark propagator used in the OPE of $\Pi(\omega)$ is

$$\langle 0 | T \{ q(x) \bar{q}(0) \} | 0 \rangle = \frac{i \not{x}}{2\pi^2 x^4} + \frac{i}{32\pi^2} \frac{\lambda^n}{2} g_s G_{\mu\nu}^n \frac{1}{x^2} (\sigma^{\mu\nu} \not{x} + \not{x} \sigma^{\mu\nu}) - \frac{\langle \bar{q}q \rangle}{12} + \dots$$

We consider the contributions of the condensates with dimension not greater than seven in our calculation. The contribution involving the quark gluon mixed condensates appears as $\alpha_s \langle \bar{q} g_s \sigma \cdot G q \rangle$. It is typically suppressed by a factor $m_0^2/(16T^2)$ compared to the quark condensate, where $m_0^2 = \langle \bar{q} g_s \sigma \cdot G q \rangle / \langle \bar{q}q \rangle = 0.8 \text{ GeV}^2$ and $T \sim 1 \text{ GeV}$. We thus ignore this contribution. The terms involving the 6-dimensional four quark condensate appear as $\alpha_s^2 \langle \bar{q}q \rangle^2$. It is of high order in α_s so it can also be omitted safely. The Feynman diagrams corresponding to the quark-level calculation are presented in Fig. 1. The Fock-Schwinger gauge $x_\mu A^\mu(x) = 0$ adopted in our calculation, together with the heavy quark propagator in Eq. (7), leads to the leading order Lagrangian of HQET $\mathcal{L}_0 = \bar{h}_v i v \cdot D h_v = \bar{h}_v i v \cdot \partial h_v$. This indicates the decoupling of the heavy quark from gauge field in the heavy quark limit and greatly simplifies our calculation.

The OPE results for $\Pi(\omega)$ read

$$\begin{aligned}
\Pi_{H_0^h}(\omega) &= \int_0^\infty dt e^{it\omega/2} \left\{ -\frac{96\alpha_s}{\pi^3} \frac{1}{t^7} - \frac{16i\alpha_s}{\pi} \langle \bar{q}q \rangle \frac{1}{t^4} - \frac{1}{4\pi^2} \langle GG \rangle \frac{1}{t^3} - \frac{1}{32\pi^2} \langle GGG \rangle \frac{1}{t} - \frac{i}{24} \langle \bar{q}q \rangle \langle GG \rangle \right\}, \\
\Pi_{M_1^h}(\omega) &= \int_0^\infty dt e^{it\omega/2} \left\{ -\frac{96\alpha_s}{\pi^3} \frac{1}{t^7} - \frac{16i\alpha_s}{\pi} \langle \bar{q}q \rangle \frac{1}{t^4} - \frac{1}{4\pi^2} \langle GG \rangle \frac{1}{t^3} + \frac{1}{64\pi^2} \langle GGG \rangle \frac{1}{t} - \frac{i}{24} \langle \bar{q}q \rangle \langle GG \rangle \right\},
\end{aligned} \tag{8}$$

and similarly

$$\Pi_{S_0^h}(\omega) = \int_0^\infty dt e^{it\omega/2} \left\{ -\frac{96\alpha_s}{\pi^3} \frac{1}{t^7} + \frac{16i\alpha_s}{\pi} \langle \bar{q}q \rangle \frac{1}{t^4} - \frac{1}{4\pi^2} \langle GG \rangle \frac{1}{t^3} - \frac{1}{32\pi^2} \langle GGG \rangle \frac{1}{t} + \frac{i}{24} \langle \bar{q}q \rangle \langle GG \rangle \right\},$$

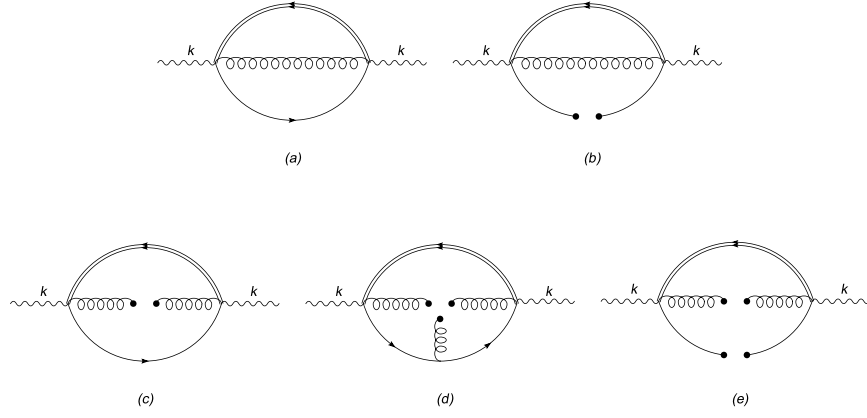


FIG. 1: The Feynman diagrams for $\Pi(\omega)$. The double solid line denotes the propagator of heavy quark Q .

$$\Pi_{T_1^h}(\omega) = \int_0^\infty dt e^{it\omega/2} \left\{ -\frac{96\alpha_s}{\pi^3} \frac{1}{t^7} + \frac{16i\alpha_s}{\pi} \langle \bar{q}q \rangle \frac{1}{t^4} - \frac{1}{4\pi^2} \langle GG \rangle \frac{1}{t^3} + \frac{1}{64\pi^2} \langle GGG \rangle \frac{1}{t} + \frac{i}{24} \langle \bar{q}q \rangle \langle GG \rangle \right\}, \quad (9)$$

where $\langle GG \rangle = \langle g_s^2 G_{\alpha\beta}^n G_{\alpha\beta}^n \rangle$, $\langle GGG \rangle = \langle g_s^3 f^{abc} G_{\alpha\beta}^a G_{\beta\gamma}^b G_{\gamma\alpha}^c \rangle$, and we used the following formulas in our calculation:

$$\begin{aligned} \langle g_s^2 G_{\alpha\beta}^m G_{\gamma\delta}^n \rangle &= \frac{\delta^{mn}}{96} [g_{\alpha\gamma} g_{\beta\delta} - g_{\alpha\delta} g_{\beta\gamma}] \langle GG \rangle, \\ \langle g_s^3 f^{abc} G_{\mu\nu}^a G_{\alpha\beta}^b G_{\rho\sigma}^c \rangle &= \frac{1}{24} \langle GGG \rangle [g_{\mu\sigma} g_{\alpha\nu} g_{\beta\rho} + g_{\mu\beta} g_{\alpha\rho} g_{\nu\sigma} + g_{\alpha\sigma} g_{\mu\rho} g_{\nu\beta} + g_{\rho\nu} g_{\mu\alpha} g_{\beta\sigma} \\ &\quad - g_{\mu\beta} g_{\alpha\sigma} g_{\rho\nu} - g_{\mu\sigma} g_{\alpha\rho} g_{\nu\beta} - g_{\alpha\nu} g_{\mu\rho} g_{\beta\sigma} - g_{\beta\rho} g_{\mu\alpha} g_{\nu\sigma}]. \end{aligned} \quad (10)$$

It turned out that $\Pi_{H_1^h}(\omega) = \Pi_{H_0^h}(\omega)$, $\Pi_{M_2^h}(\omega) = \Pi_{M_1^h}(\omega)$, $\Pi_{S_1^h}(\omega) = \Pi_{SH_0^h}(\omega)$, $\Pi_{T_2^h}(\omega) = \Pi_{T_1^h}(\omega)$, and we denote them by $\Pi_{H^h}(\omega)$, $\Pi_{M^h}(\omega)$, $\Pi_{S^h}(\omega)$, and $\Pi_{T^h}(\omega)$, respectively. This implies that the binding energy and the overlapping amplitudes of the two members of a doublet are degenerate, which is dictated by the heavy quark flavor-spin symmetry.

For $\Pi(\omega)$ we have the following dispersion relation:

$$\Pi(\omega) = \int_0^\infty \frac{\rho(s)}{s - \omega - i\epsilon} ds. \quad (11)$$

With the phenomenological spectral density $\rho_{\text{PH}} = f^2 \delta(s - 2\Lambda) + \dots$, we can rewrite the above equation to be

$$\frac{f^2}{2\Lambda - \omega} + \dots = \int_0^\infty \frac{\rho_{\text{OPE}}(s)}{s - \omega - i\epsilon} ds, \quad (12)$$

where $\Lambda = m_h - m_Q$ is the binding energy of the heavy hybrid h containing a heavy quark Q , $\rho_{\text{OPE}}(s)$ is the spectral density obtained by OPE at the quark level. By performing the Borel transformation

$$\mathcal{B}_\omega^T[f(\omega)] = \lim_{n \rightarrow \infty} \frac{(-\omega)^{n+1}}{n!} \left(\frac{d}{d\omega} \right)^n f(\omega) \Big|_{\omega = -nT}, \quad (13)$$

which is used to suppress the continuum contribution, we obtain the following sum rule with the continuum contribution subtracted:

$$f^2 e^{-2\Lambda/T} = \int_0^{s_0} \rho_{\text{OPE}}(s) e^{-s/T} ds. \quad (14)$$

It is convenient to obtain the spectral density ρ_{OPE} by performing a second Borel transformation $\mathcal{B}_{-1/T}^{1/s}[f(T)]$ to the right hand side of the above equation:

$$\rho_{H^h}(s) = \frac{\alpha_s}{3840\pi^3} s^6 - \frac{\alpha_s}{24\pi} \langle \bar{q}q \rangle s^3 + \frac{1}{32\pi^2} \langle GG \rangle s^2 - \frac{1}{32\pi^2} \langle GGG \rangle - \frac{1}{12} \langle \bar{q}q \rangle \langle GG \rangle \delta(s),$$

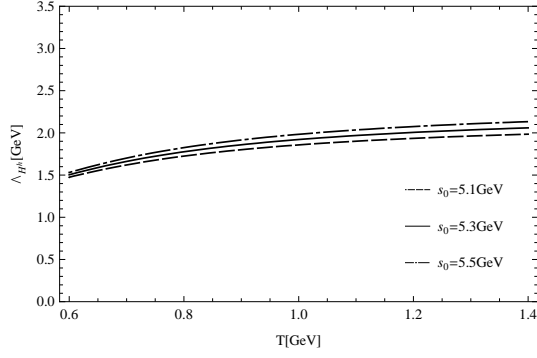


FIG. 2: The sum rule for Λ_{H^h} with continuum threshold $s_0 = 5.1, 5.3, 5.5$ GeV and the working interval $0.8 < T < 1.2$ GeV.

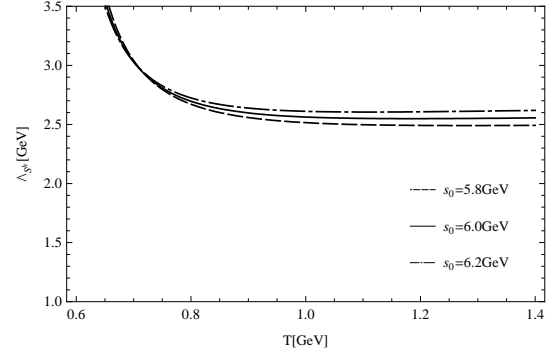


FIG. 3: The sum rule for Λ_{S^h} with continuum threshold $s_0 = 5.8, 6.0, 6.2$ GeV and the working interval $0.8 < T < 1.2$ GeV.

$$\begin{aligned}
 \rho_{M^h}(s) &= \frac{\alpha_s}{3840\pi^3} s^6 - \frac{\alpha_s}{24\pi} \langle \bar{q}q \rangle s^3 + \frac{1}{32\pi^2} \langle GG \rangle s^2 + \frac{1}{64\pi^2} \langle GGG \rangle - \frac{1}{12} \langle \bar{q}q \rangle \langle GG \rangle \delta(s), \\
 \rho_{S^h}(s) &= \frac{\alpha_s}{3840\pi^3} s^6 + \frac{\alpha_s}{24\pi} \langle \bar{q}q \rangle s^3 + \frac{1}{32\pi^2} \langle GG \rangle s^2 - \frac{1}{32\pi^2} \langle GGG \rangle + \frac{1}{12} \langle \bar{q}q \rangle \langle GG \rangle \delta(s), \\
 \rho_{T^h}(s) &= \frac{\alpha_s}{3840\pi^3} s^6 + \frac{\alpha_s}{24\pi} \langle \bar{q}q \rangle s^3 + \frac{1}{32\pi^2} \langle GG \rangle s^2 + \frac{1}{64\pi^2} \langle GGG \rangle + \frac{1}{12} \langle \bar{q}q \rangle \langle GG \rangle \delta(s).
 \end{aligned} \tag{15}$$

The binding energy and overlapping amplitudes can now be expressed as

$$\Lambda = \frac{\int_0^{s_0} s \rho(s) e^{-s/T} ds}{2 \int_0^{s_0} \rho(s) e^{-s/T} ds}, \quad f^2 = e^{2\Lambda/T} \int_0^{s_0} \rho(s) e^{-s/T} ds. \tag{16}$$

In our numerical analysis, we use $\alpha_s = 4\pi/(11 - 2/3n_f) \ln(s_0/2/\Lambda_{\text{QCD}})^2$ with $n_f = 4$ and $\Lambda_{\text{QCD}} = 220$ MeV. The mass of up and down quark is ignored. The quark condensate and gluon condensate adopt the standard values $\langle \bar{q}q \rangle = -(0.225 \text{ GeV})^3$, $\langle GG \rangle = 0.038 \text{ GeV}^4$. There are several values for the triple gluon condensate $\langle GGG \rangle = 0.045 \text{ GeV}^6$ [9], $0.06 - 0.1 \text{ GeV}^6$ [11], 0.4 GeV^6 [12]. The uncertainty caused by this difference is within 6% for Λ_{H^h} and Λ_{S^h} . So we will fix $\langle GGG \rangle$ to be 0.045 GeV^6 in the following analysis. Notice from Eq. (8) and (9) that the only difference between the OPE for H^h and M^h or S^h and T^h is the triple gluon condensate. We can then conclude that the binding energy of H^h and M^h are almost degenerate, so is the case of S^h and T^h .

From the requirement that the contribution of terms in the OPE is at least three times larger than that of the next term, except for $\langle GGG \rangle$, we get the lower limit of T denoted by T_{\min} . If we require that $T_{\max} \geq T_{\min} + 0.4 \text{ GeV}$ and the pole contribution is at least 20% of the whole sum rule, the lower limit of continuum threshold s_0 is determined.

The sum rules for Λ_{H^h} and Λ_{S^h} are plotted in Fig. 2 and Fig. 3, respectively. Finally, we list the extracted the binding energy and the overlapping amplitudes in Table I.

	Λ [GeV]	f [GeV ^{7/2}]	s_0 [GeV]
H^h/M^h	2.0	1.1	5.3
S^h/T^h	2.5	1.6	6.0

TABLE I: The values of Λ , f , and their corresponding s_0 .

IV. PIONIC COUPLINGS

To derive the sum rules for the pionic couplings of these heavy hybrids to conventional heavy mesons, we need the following interpolating currents of conventional heavy meson doublets H and S :

$$J_{H_0}^\dagger = \sqrt{\frac{1}{2}} \bar{h}_v \gamma_5 q,$$

$$\begin{aligned}
J_{H_1}^{\dagger\alpha} &= \sqrt{\frac{1}{2}} \bar{h}_v \gamma_t^\alpha q, \\
J_{S_0}^{\dagger} &= \sqrt{\frac{1}{2}} \bar{h}_v q, \\
J_{S_1}^{\dagger\alpha} &= \sqrt{\frac{1}{2}} \bar{h}_v \gamma_t^\alpha \gamma_5 q.
\end{aligned} \tag{17}$$

The overlapping amplitudes between the above currents and the corresponding heavy mesons are

$$\begin{aligned}
\langle 0 | J_{H_0}(0) | H_0(v) \rangle &= f_{H_0}, \\
\langle 0 | J_{H_1}^\alpha(0) | H_1(v, \lambda) \rangle &= f_{H_1} \epsilon_{H_1}^\alpha(v, \lambda), \\
\langle 0 | J_{S_0}(0) | S_0(v) \rangle &= f_{S_0}, \\
\langle 0 | J_{S_1}^\alpha(0) | S_1(v, \lambda) \rangle &= f_{S_1} \epsilon_{S_1}^\alpha(v, \lambda).
\end{aligned} \tag{18}$$

As an example, we present the derivation of the sum rule for the coupling constant $g_{H_1^h H_1 \pi}^p$, where p denotes the orbital momentum of the final pion. $g_{H_1^h H_1 \pi}^p$ is defined through the decay amplitude for the channel $H_1^h \rightarrow H_1 + \pi$:

$$\mathcal{M}(H_1^h \rightarrow H_1 + \pi) = I i \varepsilon^{\eta^* q v} g_{H_1^h H_1 \pi}^p, \tag{19}$$

where q is the momentum of the pion, ϵ^* is the polarization vector of the final H_1 heavy meson, the isospin factor $I = 1, 1/\sqrt{2}$ for the charged and neutral pion, respectively. $\varepsilon^{\eta^* q v} \equiv \varepsilon^{\mu\nu\rho\sigma} \eta_\mu \epsilon_\nu^* q_\rho v_\sigma$ with $\varepsilon^{\mu\nu\rho\sigma}$ the Levi-Civita tensor.

The correlation function involved in this case is:

$$i \int dx e^{-ik \cdot x} \langle \pi(q) | J_{H_1}^\beta(0) J_{H_1^h}^{\dagger\alpha}(x) | 0 \rangle = I i \varepsilon^{\alpha\beta\gamma\delta} q_\gamma v_\delta G_{H_1^h H_1 \pi}^p(\omega, \omega'), \tag{20}$$

where $\omega = 2k \cdot v$ and $\omega' = 2(k - q) \cdot v$. When $\omega, \omega' \ll 0$, we work at the quark level and express the above correlation function by the pion light-cone distribution amplitudes:

$$G_{H_1^h H_1 \pi}^p(\omega, \omega') = -\frac{1}{2} \int_0^\infty dt \int \mathcal{D}\underline{\alpha} e^{it(\frac{\bar{u}}{2}\omega + \frac{u}{2}\omega')} \frac{f_\pi m_\pi^2}{m_u + m_d} \left\{ (m_u + m_d) [2\mathcal{A}_\perp(\underline{\alpha}) - \mathcal{V}_\parallel(\underline{\alpha})] + 2\mathcal{T}(\underline{\alpha}) q \cdot v \right\}, \tag{21}$$

where $u \equiv \alpha_2 + \alpha_3$ and $\bar{u} \equiv 1 - u$. Furthermore, $G_{H_1^h H_1 \pi}^p(\omega, \omega')$ can be related to $g_{H_1^h H_1 \pi}^p$ by the dispersion relation

$$G_{H_1^h H_1 \pi}^p(\omega, \omega') = \int_0^\infty ds_1 \int_0^\infty ds_2 \frac{\rho_{H_1^h H_1 \pi}^p(s_1, s_2)}{(s_1 - \omega - i\epsilon)(s_2 - \omega' - i\epsilon)} + \int_0^\infty ds_1 \frac{\rho_1^p(s_1)}{s_1 - \omega - i\epsilon} + \int_0^\infty ds_2 \frac{\rho_2^p(s_2)}{s_2 - \omega' - i\epsilon} + \dots \tag{22}$$

with

$$\rho_{H_1^h H_1 \pi}^p(s_1, s_2) = f_{H^h} f_H g_{H_1^h H_1 \pi}^p \delta(s_1 - 2\Lambda_{H^h}) \delta(s_2 - 2\Lambda_H) + \dots \tag{23}$$

After invoking the double Borel transformation $\mathcal{B}_\omega^{T_1} \mathcal{B}_{\omega'}^{T_2}$, we extract the double dispersion relation part of Eq. (22):

$$f_{H^h} f_H g_{H_1^h H_1 \pi}^p e^{-2\bar{u}_0 \Lambda_{H^h}/T - 2u_0 \Lambda_H/T} = f_\pi m_\pi^2 \left\{ \frac{1}{m_u + m_d} \mathcal{T}^{[1]}(u_0) T^2 f_1\left(\frac{\omega_c}{T}\right) + \left[\mathcal{V}_\parallel^{[0]}(u_0) - 2\mathcal{A}_\perp^{[0]}(u_0) \right] T f_0\left(\frac{\omega_c}{T}\right) \right\}, \tag{24}$$

where

$$u_0 = \frac{T_1}{T_1 + T_2}, \quad T = \frac{T_1 T_2}{T_1 + T_2}. \tag{25}$$

The definitions of $\mathcal{F}^{[\alpha_i]}_s$ are

$$\begin{aligned}
\mathcal{F}^{[0]}(u_0) &\equiv \int_0^{u_0} \mathcal{F}(\bar{u}_0, \alpha_2, u_0 - \alpha_2) d\alpha_2, \\
\mathcal{F}^{[1]}(u_0) &\equiv \int_0^{u_0} \frac{\mathcal{F}(\bar{u}_0, \alpha_2, u_0 - \alpha_2)}{u_0 - \alpha_2} d\alpha_2 - \int_0^{\bar{u}_0} \frac{\mathcal{F}(\bar{u}_0 - \alpha_3, u_0, \alpha_3)}{\alpha_3} d\alpha_3,
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}^{[2]}(u_0) &\equiv \left. \frac{\mathcal{F}(\bar{u}_0 - \alpha_3, u_0, \alpha_3)}{\alpha_3} \right|_{\alpha_3=0} + \frac{\mathcal{F}(0, u_0, \bar{u}_0)}{\bar{u}_0} + \int_0^{u_0} d\alpha_2 \left. \frac{\partial[\mathcal{F}(1 - \alpha_2 - \alpha_3, \alpha_2, \alpha_3)/\alpha_3]}{\partial\alpha_3} \right|_{\alpha_3=u_0-\alpha_2} \\
&\quad - \int_0^{\bar{u}_0} d\alpha_3 \left. \frac{\partial[\mathcal{F}(1 - \alpha_2 - \alpha_3, \alpha_2, \alpha_3)/\alpha_3]}{\partial\alpha_2} \right|_{\alpha_2=u_0}, \\
\mathcal{F}^{[-1]}(u_0) &\equiv \int_0^1 \int_0^{1-\alpha_2} \mathcal{F}(1 - \alpha_2 - \alpha_3, \alpha_2, \alpha_3) d\alpha_3 d\alpha_2 - \int_0^{u_0} \int_0^{u_0-\alpha_2} \mathcal{F}(1 - \alpha_2 - \alpha_3, \alpha_2, \alpha_3) d\alpha_3 d\alpha_2.
\end{aligned} \tag{26}$$

The function $f_n(x)$ which is introduced while subtracting the contribution of continuum is defined as

$$f_n(x) = 1 - e^{-x} \sum_{i=0}^n \frac{x^i}{i!}. \tag{27}$$

Here we present the details of the continuum subtraction. After invoking the first double Borel transformation to the dispersion relation Eq. (22) we arrive at

$$\mathcal{B}_{\omega'}^{\frac{1}{\sigma_1}} \mathcal{B}_{\omega'}^{\frac{1}{\sigma_2}} G_{H_1^h H_1 \pi}^p(\omega, \omega') = \int_0^\infty ds_1 \int_0^\infty ds_2 e^{-s_1 \sigma_1} e^{-s_2 \sigma_2} \rho_{H_1^h H_1 \pi}(s_1, s_2). \tag{28}$$

Now the spectral density $\rho(s_1, s_2)$ can be derived after a second double Borel transformation:

$$\rho_{H_1^h H_1 \pi}(s_1, s_2) = \mathcal{B}_{-\sigma_1}^{\frac{1}{\sigma_1}} \mathcal{B}_{-\sigma_2}^{\frac{1}{\sigma_2}} \mathcal{B}_{\omega'}^{\frac{1}{\sigma_1}} \mathcal{B}_{\omega'}^{\frac{1}{\sigma_2}} G_{H_1^h H_1 \pi}^p(\omega, \omega'). \tag{29}$$

According to quark-hadron duality, we can subtract the contribution of the excited states and the continuum and arrive at

$$f_{H^h} f_H g_{H_1^h H_1 \pi}^p e^{-2\bar{u}_0 \Lambda_{H^h}/T - 2u_0 \Lambda_H/T} = \int_0^{\omega_c} ds_1 \int_0^{\omega'_c} ds_2 e^{-s_1 \sigma_1} e^{-s_2 \sigma_2} \mathcal{B}_{-\sigma_1}^{\frac{1}{\sigma_1}} \mathcal{B}_{-\sigma_2}^{\frac{1}{\sigma_2}} \mathcal{B}_{\omega'}^{\frac{1}{\sigma_1}} \mathcal{B}_{\omega'}^{\frac{1}{\sigma_2}} G_{H_1^h H_1 \pi}^p(\omega, \omega'), \tag{30}$$

where ω_c and ω'_c are the continuum thresholds of the mass rules of the H^h and H doublets, respectively. The terms of $\mathcal{B}_{\omega'}^{\frac{1}{\sigma_1}} \mathcal{B}_{\omega'}^{\frac{1}{\sigma_2}} G_{H_1^h H_1 \pi}^p(\omega, \omega')$ have general form $cu_0^m T^n = c\sigma_2^m/(\sigma_1 + \sigma_2)^{m+n}$. Here we assume $m, n > 0$ to illustrate the procedure of the continuum subtraction.

$$\begin{aligned}
&\int_0^{\omega_c} ds_1 \int_0^{\omega'_c} ds_2 e^{-s_1 \sigma_1} e^{-s_2 \sigma_2} \mathcal{B}_{-\sigma_1}^{\frac{1}{\sigma_1}} \mathcal{B}_{-\sigma_2}^{\frac{1}{\sigma_2}} \frac{\sigma_2^m}{(\sigma_1 + \sigma_2)^{m+n}} \\
&= \int_0^{\omega_c} ds_1 \int_0^{\omega'_c} ds_2 e^{-s_1 \sigma_1} e^{-s_2 \sigma_2} \frac{1}{\Gamma(m+n)} \left[-\frac{\partial \delta(s_1 - s_2)}{\partial s_1} \right]^m s_1^{m+n-1} \\
&= 2 \int_0^{\omega'_c} ds_+ \int_{-s_+}^{s_+} ds_- e^{-s_+ T} e^{s_- T} \frac{(s_+ - s_-)^{m+n-1}}{2^m \Gamma(m+n)} \left(\frac{\partial}{\partial s_-} \right)^m \delta(2s_-) \\
&= \frac{T^n}{2^m} \sum_{i=0}^m \frac{m!}{i!(m-i)!} (2u_0 - 1)^i f_{n-1+i}(\frac{\omega'_c}{T}),
\end{aligned} \tag{31}$$

where $s_+ = (s_1 + s_2)/2$, $s_- = (s_2 - s_1)/2$, $1/T_- = 1/T_1 - 1/T_2$ and we assume $\omega_c > \omega'_c$. We will work at the symmetry point, namely $T_1 = T_2 = 2T$ and $u_0 = 1/2$. This leads to a greatly simplified continuum subtraction:

$$\begin{aligned}
u_0^m T^n &\rightarrow \frac{T^n}{2^m} \sum_{i=0}^m \frac{m!}{i!(m-i)!} (2u_0 - 1)^i f_{n-1+i}(\frac{\omega'_c}{T}) \\
&= u_0^m T^n f_{n-1}(\frac{\omega'_c}{T}),
\end{aligned} \tag{32}$$

namely $T^n \rightarrow T^n f_{n-1}(\omega'_c/T)$. This choice of u_0 is based on the consideration that the working interval of the Borel parameter T of the mass sum rules for H and S is about $0.8 < T < 1.1$ GeV[13], which is very close to that of the mass sum rules for D^h ($D = H/S/M/T$). This will enable us to subtract the continuum contribution cleanly, while the asymmetric choice will lead to the very difficult continuum subtraction [15].

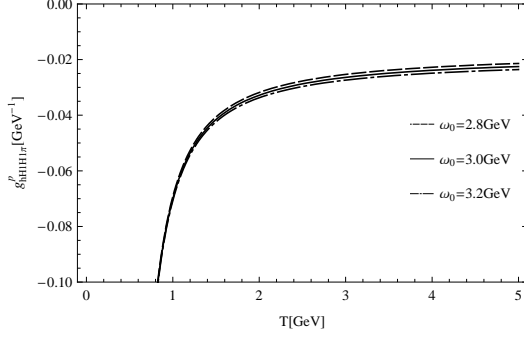


FIG. 4: The sum rule for $g_{H^h H^1 \pi}^p$ with continuum threshold $\omega'_c = 2.8, 3.0, 3.2$ GeV.

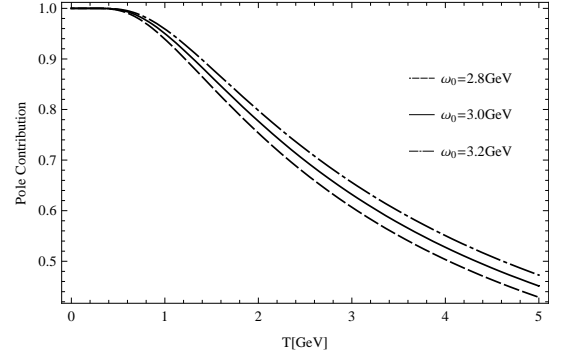


FIG. 5: The determination of the upper limit of T with continuum threshold $\omega'_c = 2.8, 3.0, 3.2$ GeV.

The binding energy and the overlapping amplitudes extracted in the previous section are used in our numerical analysis of the sum rules for the above pionic couplings, together with the following values for $\Lambda_{H/S}$ and $f_{H/S}$ [13]:

$$\begin{aligned} \Lambda_H &= 0.50 \text{ GeV}, & f_H &= 0.25 \text{ GeV}^{3/2}, \\ \Lambda_S &= 1.15 \text{ GeV}, & f_S &= 0.40 \text{ GeV}^{3/2}. \end{aligned}$$

In this work the π decay constant is taken to be $f_\pi = 131$ MeV. $\mu_\pi \equiv m_\pi^2/(m_u + m_d) = (1.573 \pm 0.174)$ GeV is given in Ref. [14]. The parameters appearing in the π distribution amplitudes are listed below [14]. We use the values at the scale $\mu = 1$ GeV in our calculation.

a_2	η_3	ω_3	η_4	ω_4	h_{00}	v_{00}	a_{10}	v_{10}	h_{01}	h_{10}
0.25	0.015	-1.5	10	0.2	-3.33	-3.33	5.14	5.25	3.46	7.03

From the requirement of the stability of the coupling constant to the variation of the Borel parameter T and the requirement that the pole contribution is larger than 40%, we get the working interval of $T_{min} < T < T_{max}$, which is plotted in Fig. 5. The resulting sum rule is plotted with $\omega'_c = 2.8, 3.0, 3.2$ GeV in Fig. 4.

Similarly, we have

$$\begin{aligned} f_{S^h} f_S g_{S_1^h S_1 \pi}^p e^{-2\bar{u}_0 \Lambda_{S^h}/T - 2u_0 \Lambda_S/T} &= f_\pi m_\pi^2 \left\{ \frac{1}{m_u + m_d} \mathcal{T}^{[1]}(u_0) T^2 f_1\left(\frac{\omega'_c}{T}\right) - \left[\mathcal{V}_{\parallel}^{[0]}(u_0) - 2\mathcal{A}_{\perp}^{[0]}(u_0) \right] T f_0\left(\frac{\omega'_c}{T}\right) \right\}, \\ f_{M^h} f_H g_{M_1^h H_1 \pi}^p e^{-2\bar{u}_0 \Lambda_{M^h}/T - 2u_0 \Lambda_H/T} &= \frac{1}{\sqrt{2}} f_\pi m_\pi^2 \left\{ \frac{1}{2(m_u + m_d)} \mathcal{T}^{[1]}(u_0) T^2 f_1\left(\frac{\omega'_c}{T}\right) - \left[\mathcal{A}_{\perp}^{[0]}(u_0) + \mathcal{V}_{\parallel}^{[0]}(u_0) \right] T f_0\left(\frac{\omega'_c}{T}\right) \right\}, \\ f_{T^h} f_S g_{T_1^h S_1 \pi}^p e^{-2\bar{u}_0 \Lambda_{T^h}/T - 2u_0 \Lambda_S/T} &= \frac{1}{\sqrt{2}} f_\pi m_\pi^2 \left\{ \frac{1}{2(m_u + m_d)} \mathcal{T}^{[1]}(u_0) T^2 f_1\left(\frac{\omega'_c}{T}\right) + \left[\mathcal{A}_{\perp}^{[0]}(u_0) + \mathcal{V}_{\parallel}^{[0]}(u_0) \right] T f_0\left(\frac{\omega'_c}{T}\right) \right\}, \\ f_{M^h} f_S g_{M_1^h S_1 \pi}^d e^{-2\bar{u}_0 \Lambda_{M^h}/T - 2u_0 \Lambda_S/T} &= -\frac{3}{\sqrt{2}} f_\pi m_\pi^2 \left\{ \frac{1}{m_u + m_d} \mathcal{T}^{[0]}(u_0) T f_0\left(\frac{\omega'_c}{T}\right) - 2 \left[\mathcal{V}_{\perp}^{[-1]}(u_0) + \mathcal{V}_{\parallel}^{[-1]}(u_0) \right] \right\}, \\ f_{T^h} f_H g_{T_1^h H_1 \pi}^d e^{-2\bar{u}_0 \Lambda_{T^h}/T - 2u_0 \Lambda_H/T} &= -\frac{3}{\sqrt{2}} f_\pi m_\pi^2 \left\{ \frac{1}{m_u + m_d} \mathcal{T}^{[0]}(u_0) T f_0\left(\frac{\omega'_c}{T}\right) + 2 \left[\mathcal{V}_{\perp}^{[-1]}(u_0) + \mathcal{V}_{\parallel}^{[-1]}(u_0) \right] \right\}, \\ f_{H^h} f_S g_{H_1^h S_1 \pi}^s e^{-2\bar{u}_0 \Lambda_{H^h}/T - 2u_0 \Lambda_S/T} &= \frac{1}{2} f_\pi m_\pi^2 \left\{ \frac{1}{m_u + m_d} \mathcal{T}^{[2]}(u_0) T^3 f_2\left(\frac{\omega'_c}{T}\right) + \left[2\mathcal{V}_{\perp}^{[1]}(u_0) - \mathcal{V}_{\parallel}^{[1]}(u_0) \right] T^2 f_1\left(\frac{\omega'_c}{T}\right) \right. \\ &\quad \left. + \frac{4m_\pi^2}{m_u + m_d} \mathcal{T}^{[0]}(u_0) T f_0\left(\frac{\omega'_c}{T}\right) + 4m_\pi^2 \left[\mathcal{V}_{\perp}^{[-1]}(u_0) + \mathcal{V}_{\parallel}^{[-1]}(u_0) \right] \right\}, \\ f_{S^h} f_H g_{S_1^h H_1 \pi}^s e^{-2\bar{u}_0 \Lambda_{S^h}/T - 2u_0 \Lambda_H/T} &= \frac{1}{2} f_\pi m_\pi^2 \left\{ \frac{1}{m_u + m_d} \mathcal{T}^{[2]}(u_0) T^3 f_2\left(\frac{\omega'_c}{T}\right) - \left[2\mathcal{V}_{\perp}^{[1]}(u_0) - \mathcal{V}_{\parallel}^{[1]}(u_0) \right] T^2 f_1\left(\frac{\omega'_c}{T}\right) \right. \\ &\quad \left. + \frac{4m_\pi^2}{m_u + m_d} \mathcal{T}^{[0]}(u_0) T f_0\left(\frac{\omega'_c}{T}\right) - 4m_\pi^2 \left[\mathcal{V}_{\perp}^{[-1]}(u_0) + \mathcal{V}_{\parallel}^{[-1]}(u_0) \right] \right\}. \end{aligned}$$

(33)

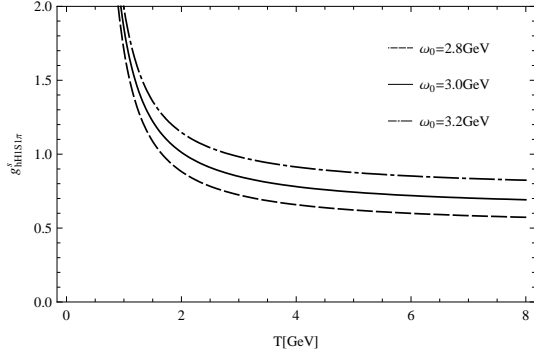


FIG. 6: The sum rule for $g_{H_1^h H_1 \pi}^p$ with continuum threshold $\omega'_c = 2.8, 3.0, 3.2$ GeV.

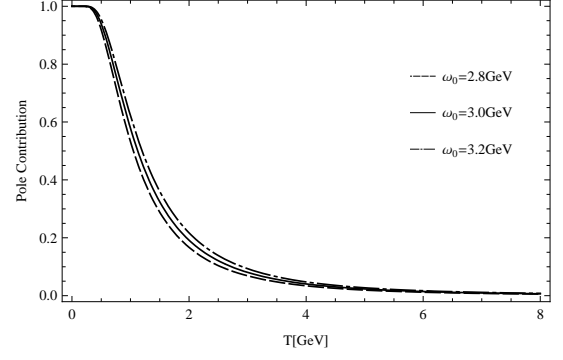


FIG. 7: The upper limit of T which is not larger than its lower limit, indicates the nonexistence of a stable working interval of T ($\omega'_c = 2.8, 3.0, 3.2$ GeV).

We notice that

$$\begin{aligned}
g_{H^h H \pi}^p &\equiv g_{H_1^h H_1 \pi}^p = -g_{H_0^h H_1 \pi}^p = -g_{H_1^h H_0 \pi}^p, \\
g_{H^h S \pi}^s &\equiv g_{H_1^h S_1 \pi}^s = -g_{H_0^h S_0 \pi}^s, \\
g_{S^h H \pi}^s &\equiv g_{S_1^h H_1 \pi}^s = g_{S_0^h H_0 \pi}^s, \\
g_{S^h S \pi}^p &\equiv g_{S_1^h S_1 \pi}^p = g_{S_0^h S_0 \pi}^p = -g_{S_1^h S_0 \pi}^p, \\
g_{M^h H \pi}^p &\equiv g_{M_1^h H_1 \pi}^p = \frac{1}{2}g_{M_1^h H_0 \pi}^p = -\sqrt{\frac{2}{3}}g_{M_2^h H_1 \pi}^p, \\
g_{M^h S \pi}^d &\equiv g_{M_1^h S_1 \pi}^d = \sqrt{\frac{3}{2}}g_{M_2^h S_0 \pi}^d = -\sqrt{6}g_{M_2^h S_1 \pi}^d, \\
g_{T^h H \pi}^d &\equiv g_{T_1^h H_1 \pi}^d = -\sqrt{\frac{3}{2}}g_{T_2^h H_0 \pi}^d = \sqrt{6}g_{T_2^h H_1 \pi}^d, \\
g_{T^h S \pi}^p &\equiv g_{T_1^h S_1 \pi}^p = \frac{1}{2}g_{T_1^h S_0 \pi}^p = \frac{1}{\sqrt{6}}g_{T_2^h S_1 \pi}^p.
\end{aligned} \tag{34}$$

These relations are consistent with the heavy quark flavor-spin symmetry.

From Fig. 6 and Fig. 7 we can see that there is almost no stable working interval of the Borel parameter T , namely the sum rule for $g_{H^h S \pi}^s$ is unstable, so is the case of $g_{S^h H \pi}^s$. Despite this, we make crude estimates for them, and use these estimates to calculate the partial widths of the corresponding decay channels.

$g_{H_1^h H_1 \pi}^p$	$g_{S_1^h S_1 \pi}^p$	$g_{H_1^h S_1 \pi}^s$	$g_{S_1^h H_1 \pi}^s$	$g_{M_1^h H_1 \pi}^p$	$g_{T_1^h S_1 \pi}^p$	$g_{M_1^h S_1 \pi}^d$	$g_{T_1^h H_1 \pi}^d$
0.03	0.01	0.7*	0.7*	0.2	0.1	0.08	0.1

TABLE II: The absolute values of the coupling constants. The units of the P - and D -wave coupling constants are GeV^{-1} and GeV^{-2} , respectively. Here the superscript * indicates the instability of the sum rule for that coupling constant.

The extracted coupling constants and the partial widths obtained with them are collected in Table II and Table III, respectively. These numerical values are rather small as a whole. The annihilation of the gluon degree of freedom in the decay processes may be responsible for these weak couplings. The tensor structures of the involved decay channels are also included in Table III, where the tensor structures of various partial waves are defined as

$$\begin{aligned}
T_s &= 1, \\
T_p^\alpha &= q_t^\alpha,
\end{aligned}$$

	H_0^h	H_1^h	S_0^h	S_1^h	M_1^h	M_2^h	T_1^h	T_2^h
$\rightarrow H_0^0 + \pi^+$		$\ll 0.1$	14.1/24.4		1.9/3.9			0.1/0.4
		T_p^α	T_s		T_p^α			$T_d^{\alpha_1\alpha_2}$
$\rightarrow H_1^0 + \pi^+$	0.03/0.05	$\ll 0.1$		17.0/25.2	0.9/1.9	0.8/1.5	0.4/1.1	0.2/0.6
	T_p^β	$T_p^{\alpha\beta}$		T_s	$T_p^{\alpha\beta}$	$T_p^{\alpha_1\alpha_2\beta}$	$T_d^{\alpha\beta}$	$T_d^{\alpha_1\alpha_2\beta}$
$\rightarrow S_0^0 + \pi^+$	10.7/13.4			$\ll 0.1$		$\ll 0.1$	0.5/0.8	
	T_s			T_p^α		$T_d^{\alpha_1\alpha_2}$	T_p^α	
$\rightarrow S_1^0 + \pi^+$		11.0/13.5	$\ll 0.1$	$\ll 0.1$	$\ll 0.1$	$\ll 0.1$	0.2/0.4	0.8/1.2
		T_s	T_p^β	$T_p^{\alpha\beta}$	$T_d^{\alpha\beta}$	$T_d^{\alpha_1\alpha_2\beta}$	$T_p^{\alpha\beta}$	$T_p^{\alpha_1\alpha_2\beta}$

TABLE III: The partial widths of the decay modes $D^h/B^h \rightarrow D/B + \pi$ in unit of MeV, together with the tensor structures of these decay channels in the heavy quark limit. The masses of the c quark and b quark used in the calculation are 1.4 GeV and 4.8 GeV, respectively.

$$\begin{aligned}
T_p^{\alpha\beta} &= i\varepsilon^{\alpha\beta qv}, \\
T_p^{\alpha_1\alpha_2\beta} &= \frac{1}{2}g_t^{\alpha_1\beta}q_t^{\alpha_2} + \frac{1}{2}g_t^{\alpha_2\beta}q_t^{\alpha_1} - \frac{1}{3}g_t^{\alpha_1\alpha_2}q_t^\beta, \\
T_d^{\alpha_1\alpha_2} &= q_t^{\alpha_1}q_t^{\alpha_2} - \frac{1}{3}g_t^{\alpha_1\alpha_2}q_t^2, \\
T_d^{\alpha_1\alpha_2\beta} &= i\varepsilon^{\beta\alpha_1qv}q_t^{\alpha_2} + i\varepsilon^{\beta\alpha_2qv}q_t^{\alpha_1}.
\end{aligned} \tag{35}$$

V. CONCLUSION

We constructed the appropriate interpolating currents for the hybrid mesons containing one heavy quark($q\bar{Q}g$). Then we calculated the binding energy and the pionic couplings at the leading order of HQET within the framework of LCQSR. The mass sum rules and most of the sum rules for the pionic couplings are stable with the variations of the Borel parameter and the continuum threshold. For the sum rules for $g_{H^h S\pi}^s$ and $g_{S^h H\pi}^s$, we can not find a stable working interval of T . We found that the binding energy of the heavy hybrid H^h and M^h are degenerate, so is the case of S^h and T^h . As far as the pionic couplings are concerned, the extracted couplings are rather small as a whole.

Some possible sources of the errors in our calculation include the inherent inaccuracy of SVZ sum rules and LCQSR: the omission of the higher dimensional condensates, in our case the quark gluon mixed condensates etc. in the mass sum rules, and the higher twist terms in the OPE near the light-cone, the variation of the binding energy and the coupling constant with the continuum threshold ω_c and the Borel parameter T in the working interval, the omission of the higher conformal partial waves in the light-cone distribution amplitudes of pion, and the uncertainty in the parameters that appear in these light-cone distribution amplitudes. The uncertainty in f 's and $\bar{\Lambda}$'s is another source of errors to the light-cone sum rules. Finally, the $1/m_Q$ correction may turn out to be quite large concerning the charm quark while such a correction is under control in the case of the bottom quark.

The weak pionic couplings between the heavy hybrid mesons($q\bar{Q}g$) and the conventional $q\bar{Q}$ systems render narrow partial widths of the corresponding decay channels. We have made a rough estimate of these partial widths. These heavy hybrid mesons are found to be quite narrow with a width around several tens MeV in the heavy quark limit. We hope that this estimate, together with the calculation on the binding energy of the heavy hybrid doublets, may be helpful to the future experimental search of these unconventional heavy mesons.

Acknowledgments

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Appendix A: The light-cone distribution amplitudes of the pion

The 2-particle distribution amplitudes of the π meson are defined as [14]

$$\begin{aligned}
 \langle 0 | \bar{u}(z) \gamma_\mu \gamma_5 d(-z) | \pi^-(P) \rangle &= i f_\pi p_\mu \int_0^1 du e^{i\xi p z} \phi_\pi(u) + \frac{i}{2} f_\pi m^2 \frac{1}{p z} z_\mu \int_0^1 du e^{i\xi p z} g_\pi(u), \\
 \langle 0 | \bar{u}(z) i \gamma_5 d(-z) | \pi(P) \rangle &= \frac{f_\pi m_\pi^2}{m_u + m_d} \int_0^1 du e^{i\xi p z} \phi_p(u), \\
 \langle 0 | \bar{u}(z) \sigma_{\alpha\beta} \gamma_5 d(-z) | \pi(P) \rangle &= -\frac{i}{3} \frac{f_\pi m_\pi^2}{m_u + m_d} (p_\alpha z_\beta - p_\beta z_\alpha) \int_0^1 du e^{i\xi p z} \phi_\sigma(u),
 \end{aligned} \tag{A1}$$

where $\xi \equiv 2u - 1$, ϕ_π is the leading twist-2 distribution amplitude, $\phi_{(p,\sigma)}$ are of twist-3. All the above distribution amplitudes $\phi = \{\phi_\pi, \phi_p, \phi_\sigma, g_\pi\}$ are normalized to unity: $\int_0^1 du \phi(u) = 1$.

There is one 3-particle distribution amplitudes of twist-3, defined as [14]

$$\langle 0 | \bar{u}(z) \sigma_{\mu\nu} \gamma_5 g_s G_{\alpha\beta}(vz) d(-z) | \pi^-(P) \rangle = i \frac{f_\pi m_\pi^2}{m_u + m_d} (p_\alpha p_\mu g_{\nu\beta}^\perp - p_\alpha p_\nu g_{\mu\beta}^\perp - p_\beta p_\mu g_{\nu\alpha}^\perp + p_\beta p_\nu g_{\mu\alpha}^\perp) \mathcal{T}(v, pz), \tag{A2}$$

where we used the following notation for the integral defining the 3-particle distribution amplitude:

$$\mathcal{T}(v, pz) = \int \mathcal{D}\underline{\alpha} e^{-ipz(\alpha_u - \alpha_d + v\alpha_g)} \mathcal{T}(\alpha_d, \alpha_u, \alpha_g). \quad (\text{A3})$$

Here $\underline{\alpha}$ is the set of three momentum fractions α_d , α_u , and α_g . The integration measure is

$$\int \mathcal{D}\underline{\alpha} = \int_0^1 d\alpha_d d\alpha_u d\alpha_g \delta(1 - \alpha_u - \alpha_d - \alpha_g). \quad (\text{A4})$$

The 3-particle distribution amplitudes of twist-4 are

$$\begin{aligned} \langle 0 | \bar{u}(z) \gamma_\mu \gamma_5 g_s G_{\alpha\beta}(vz) d(-z) | \pi^-(P) \rangle &= p_\mu (p_\alpha z_\beta - p_\beta z_\alpha) \frac{1}{pz} f_\pi m_\pi^2 \mathcal{A}_\parallel(v, pz) + (p_\beta g_{\alpha\mu}^\perp - p_\alpha g_{\beta\mu}^\perp) f_\pi m_\pi^2 \mathcal{A}_\perp(v, pz), \\ \langle 0 | \bar{u}(z) \gamma_\mu i g_s \tilde{G}_{\alpha\beta}(vz) d(-z) | \pi^-(P) \rangle &= p_\mu (p_\alpha z_\beta - p_\beta z_\alpha) \frac{1}{pz} f_\pi m_\pi^2 \mathcal{V}_\parallel(v, pz) + (p_\beta g_{\alpha\mu}^\perp - p_\alpha g_{\beta\mu}^\perp) f_\pi m_\pi^2 \mathcal{V}_\perp(v, pz), \end{aligned} \quad (\text{A5})$$

where $\tilde{G}_{\alpha\beta}$ is the dual field $\tilde{G}_{\alpha\beta} \equiv \frac{1}{2} \varepsilon_{\alpha\beta\gamma\delta} G^{\gamma\delta}$.

We also use the distribution amplitude given in Ref. [14]:

$$\phi_\pi(u) = 6u(1-u) \left(1 + a_2 C_2^{3/2}(\xi) \right), \quad (\text{A6})$$

$$g_\pi(u) = 1 + \left(1 + \frac{18}{7} a_2 + 60\eta_3 + \frac{20}{3} \eta_4 \right) C_2^{1/2}(\xi) + \left(-\frac{9}{28} a_2 - 6\eta_3 \omega_3 \right) C_4^{1/2}(\xi), \quad (\text{A7})$$

$$\begin{aligned} \mathbb{A}(u) &= 6u\bar{u} \left\{ \frac{16}{15} + \frac{24}{35} a_2 + 20\eta_3 + \frac{20}{9} \eta_4 \right. \\ &\quad + \left(-\frac{1}{15} + \frac{1}{16} - \frac{7}{27} \eta_3 \omega_3 - \frac{10}{27} \eta_4 \right) C_2^{3/2}(\xi) + \left(-\frac{11}{210} a_2 - \frac{4}{135} \eta_3 \omega_3 \right) C_4^{3/2}(\xi) \Big\} \\ &\quad + \left(-\frac{18}{5} a_2 + 21\eta_4 \omega_4 \right) \{ 2u^3(10 - 15u + 6u^2) \ln u + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2) \ln \bar{u} + u\bar{u}(2 + 13u\bar{u}) \}, \end{aligned} \quad (\text{A8})$$

$$\mathbb{B}(u) = g_\pi(u) - \phi_\pi(u), \quad (\text{A9})$$

$$\phi_p(u) = 1 + \left(30\eta_3 - \frac{5}{2} \rho_\pi^2 \right) C_2^{1/2}(\xi) + \left(-3\eta_3 \omega_3 - \frac{27}{20} \rho_\pi^2 - \frac{81}{10} \rho_\pi^2 a_2 \right) C_4^{1/2}(\xi), \quad (\text{A10})$$

$$\phi_\sigma(u) = 6u(1-u) \left\{ 1 + \left(5\eta_3 - \frac{1}{2} \eta_3 \omega_3 - \frac{7}{20} \rho_\pi^2 - \frac{3}{5} \rho_\pi^2 a_2 \right) C_2^{3/2}(\xi) \right\}, \quad (\text{A11})$$

$$\mathcal{T}(\underline{\alpha}) = 360\eta_3 \alpha_u \alpha_d \alpha_g^2 \left\{ 1 + \omega_3 \frac{1}{2} (7\alpha_g - 3) \right\}, \quad (\text{A12})$$

$$\mathcal{V}_\parallel(\underline{\alpha}) = 120\alpha_u \alpha_d \alpha_g (v_{00} + v_{10}(3\alpha_g - 1)),$$

$$\mathcal{A}_\parallel(\underline{\alpha}) = 120\alpha_u \alpha_d \alpha_g a_{10} (\alpha_d - \alpha_u), \quad (\text{A13})$$

$$\mathcal{V}_\perp(\underline{\alpha}) = -30\alpha_g^2 \left[h_{00}(1 - \alpha_g) + h_{01} [\alpha_g(1 - \alpha_g) - 6\alpha_u \alpha_d] + h_{10} \left[\alpha_g(1 - \alpha_g) - \frac{3}{2} (\alpha_u^2 + \alpha_d^2) \right] \right], \quad (\text{A14})$$

$$\mathcal{A}_\perp(\underline{\alpha}) = 30\alpha_g^2 (\alpha_u - \alpha_d) \left[h_{00} + h_{01} \alpha_g + \frac{1}{2} h_{10} (5\alpha_g - 3) \right], \quad (\text{A15})$$

where $C_n^m(\xi)$ are Gegenbauer polynomials.

The definitions and the specific forms of the η light-cone distribution amplitudes adopted in the text are similar to those of the pion. For more details see Ref. [14].